UNIT- 6

OSCILLATIONS AND WAVES

SIMPLE HARMONIC MOTION -:

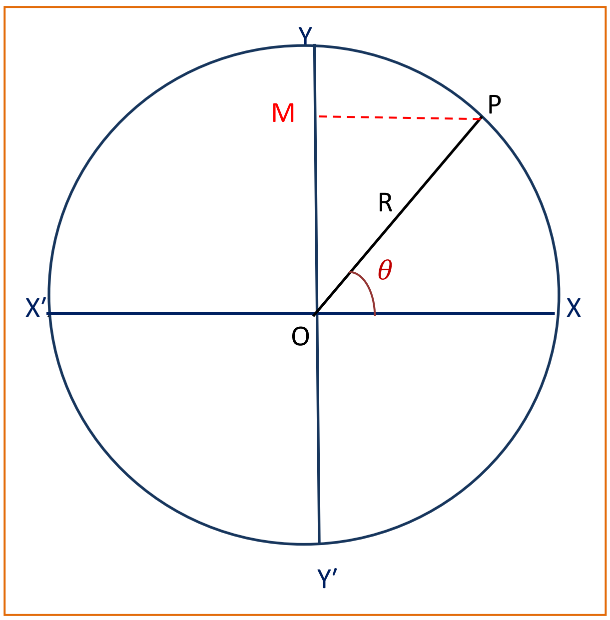
- It is a special type of periodic motion in which a particle moves to and fro about a mean position or equilibrium position under a restoring force which is always directed towards the mean position and whose magnitude at any instant is directly proportional to the displacement of the particle from the mean position at that instant.
- If a particle executing s.h.m. has a displacement 'y' at any instant under a restoring force, then the magnitude of the restoring force at that instant is given by

$$F = -ky$$

where 'k' is known as the force constant and its S.I. unit is N/m.

- Negative sign in the above equation indicates that the restoring force is always directed towards the mean position.

GEOMETRICAL INTERPRETATION OF S.H.M. -:

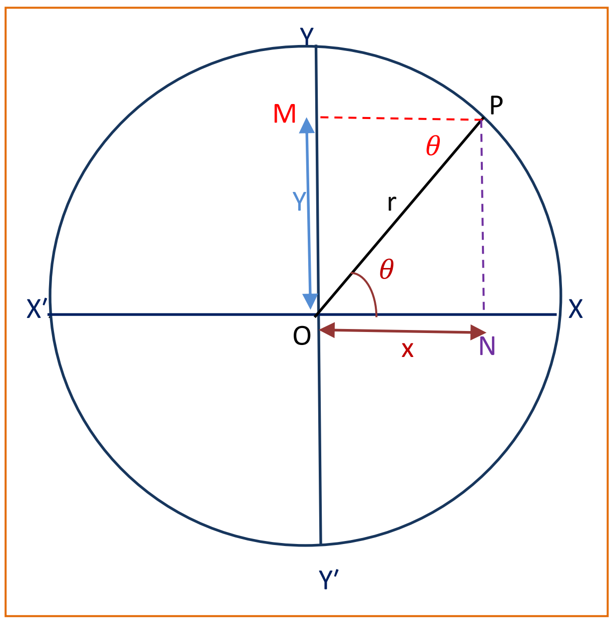


- Suppose a particle is moving along the circumference of a circle of radius 'R' and center at 'O' with uniform angular velocity.
- The particle is called as the reference particle and the circle along which the particle is moving is called as the circle of reference.
- Let XOX' and YOY' are two mutually perpendicular diameters of the circle of reference.
- Let at any instant of time the particle is at 'P' and the perpendicular drawn from 'P' on the diameter YOY' is at 'M'.
- So 'M' is called as the projection or the foot of the perpendicular drawn from 'P' on the diameter YOY'.
- When the particle moves from X to Y, its projection M moves from O to Y on the diameter YOY'.

Similarly,

- When the particle moves from Y to X′, its projection M moves from Y to O on the diameter YOY′.
- When the particle moves from X′ to Y′, its projection M moves from O to Y′ on the diameter YOY′.
- When the particle moves from **Y′** to X, its projection M moves from Y′ to O on the diameter YOY′.
- So during the time the particle goes once around the circle and completes one revolution. At the same time its projection ‘M’ moves to and fro about the center of the reference circle ‘O’ on the diameter YOY′ and completes one vibration.
- So the motion of the projection ‘M’ on the diameter YOY is performing simple harmonic motion about the center ‘O’ of the circle of reference as the mean position.
- So simple harmonic motion can also be defined as the projection of uniform circular motion on any diameter of circle of reference.

CHARACTERISTICS OF S.H.M. -:



1) DISPLACEMENT -:

- The displacement of a particle executing S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.
- Let the reference particle ‘P’ moves along the circumference of a reference circle of radius ‘r’ with uniform angular velocity (ω).
- Let the reference particles starts from ‘X’ and traces an angle ‘θ’ as it reaches at ‘P’ in a time ‘t’ second.
- We know that $\boldsymbol{angular\ velocity\ (\omega)} = \frac{\boldsymbol{angular\ displacement\ (\theta)}}{\boldsymbol{time\ (t)}}$

$$\boldsymbol{\omega = \frac{\theta}{t}}$$

$$\Rightarrow \boldsymbol{\theta = \omega t} \quad \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_(1)$$

- The projection drawn from ‘P’ on the diameter YOY’ is at M.
- So here displacement of the particle in S.H.M is OM.

  Let $OM = y$

- In the ⊿ OPM, $\sin\theta = \frac{OM}{OP}$

$$\Rightarrow \sin\theta = \frac{y}{r}$$

$$\Rightarrow y = r\sin\theta$$

$$\boxed{\Rightarrow \boldsymbol{y = r\sin\omega t}}$$ ______________(2) [using equ. (1)]

So this the expression for displacement of particle executing S.H.M.

2) VELOCITY -:

- The velocity of a particle executing S.H.M. at any instant is defined as the time rate of change of displacement of the particle at that instant.
- So $velovity(v) = \frac{d}{dt}(y)$

$$\Rightarrow v = \frac{d}{dt}(r\sin\omega t)$$

$$\Rightarrow v = r\frac{d}{dt}(\sin\omega t)$$

$$\Rightarrow \boldsymbol{v = r\omega\cos\omega t}$$ _______________ (3)

$$\Rightarrow v = r\omega\sqrt{(1 - sin^2\omega t)}$$

$$\Rightarrow v = r\omega\sqrt{\left(1 - \frac{y^2}{r^2}\right)} \quad [using\ equ.(2)]$$

$$\boxed{\Rightarrow \boldsymbol{v = \omega\sqrt{(r^2 - y^2)}}}$$ ________________ (4)

So this is the expression for the velocity of a particle executing S.H.M.

- At mean position, $y = 0$

  Then $v = \omega r$ (maximum velocity)

- At extreme position, $y = r$

  Then $v = 0$ (zero velocity)

- So the velocity of a particle in S.H.M is not uniform throughout the motion. It is maximum at the mean position is zero (minimum) at the extreme position.

3) ACCELERATION -:

- The acceleration of a particle executing S.H.M. at any instant is defined as the time rate of change of velocity of the particle at that instant.
- So $acceleration\ (a) = \frac{d}{dt}(v)$

$$\Rightarrow a = \frac{d}{dt}(r\omega\cos\omega t)$$

$$\Rightarrow a = r\omega\frac{d}{dt}(\cos\omega t)$$

$$\Rightarrow a = r\omega(-\omega \sin \omega t)$$
$$\Rightarrow a = -\omega^2 (r \sin \omega t)$$
$$\boxed{\Rightarrow \boldsymbol{a = -\omega^2 y}} \qquad \text{\_\_\_\_\_\_\_\_\_\_ (5)}$$

So this is the expression for the acceleration of a particle executing S.H.M.

- At mean position, $y = 0$

  Then a$=$ 0 (zero acceleration)
- At extreme position, $y = r$

  Then $a = -\omega^2 r$ (maximum acceleration)
- So the acceleration of a particle in S.H.M is not uniform throughout the motion. It is maximum at the extreme position and is minimum at the mean position.
- Above equation indicates that the acceleration of a particle in S.H.M is directly proportional to the displacement of the particle i.e. $a \alpha y$.
- Negative sign in the above equation indicates that acceleration is always directed opposite to one in which displacement increases. Since displacement increases away from the mean position, so the acceleration is always directed towards the mean position in S.H.M.

<u>WAVE MOTION -:</u>

- It is a form of disturbance that travels through a medium due to repeated periodic motion of the particles in the medium and the motion being transferred from one particle to another without the actual motion of the particles in the medium.
- In a wave motion the energy and momentum are transferred one particle to another in the medium.

TYPES OF WAVE -:

1) **MECHANICAL/ELASTIC WAVE -:** In this type of wave propagation material medium is necessary. Ex- sound wave, water wave etc.
2) **NON-MECHANICAL/ NON-ELASTIC WAVE -:** It is a type of wave which does not required any medium for its propagation. Ex- light wave.
3) **MATTER WAVE -:** The waves associated with the fundamental particles like electron, proton, neutron are called matter waves or De-Broglie waves.

 # According to the vibration of the particles in the medium with respect to the direction of wave propagation there are two types of waves.

<u>LONGITUDINALWAVE -:</u>

- In this type of wave motion particles in the medium vibrate parallel to the direction of wave propagation.
- During the propagation of this type of wave some particles of the medium come close together forming compression.
- At other places the particles of the medium move further apart from each other forming rarefaction.
- Material medium is necessary for the propagation of this type of wave.

- Particle density varies throughout the medium during the propagation of the wave and it is maximum at compression region and is minimum at rarefaction region.
- Example – sound wave.

TRANSVERSE WAVE -:

- In this type of wave the displacement of the particles in the medium is perpendicular to the direction of propagation of the wave.
- During the propagation of this type of wave some layer of particle in the medium gets raised above the normal level and is called as the crest.
- At other places some layers of particles in the medium gets depressed below the normal level and is called as the trough.
- Material medium may or may not be essential for the propagation of this type of wave.
- Particle density does not throughout the medium during the propagation of the wave.
- Example – light wave, water wave.

LONGITUDINAL WAVE	TRANSVERSE WAVE
1. In this type of wave the displacement of the particles in the medium is parallel to the direction of propagation of the wave. 2. Compressions and rarefactions are formed during the propagation of the wave in the medium. 3. Pressure varies throughout the medium and is maximum at compression and is minimum at rarefaction. 4. During the propagation of this type of wave there is a temporary change in the size of the medium. 5. Example- sound wave.	1. In this type of wave the displacement of the particles in the medium is perpendicular to the direction of propagation of the wave. 2. Crests and troughs are formed during the propagation of the wave in the medium. 3. There is no change in the pressure and particle density of the medium during the propagation of the wave. 4. During the propagation of this type of wave there is a temporary change in the shape of the medium. 5. Example- light wave, water wave.

CHARACTERISTICS OF A WAVE -:

- **AMPLITUDE (A)-:**
  - It is the maximum displacement suffered by the particles of the medium from the mean position of rest when a wave travels through the medium.
  - S.I. unit – meter (m)
  - Dimension – $[L^1]$

- **WAVELENGTH (λ) -:**
  - It is the distance between any two consecutive particles in the medium which are in the same state of vibration.

OR

  - During the time the particle of the medium completes one vibration the distance travelled by the wave is equal to its wavelength.
  - S.I. unit – meter (m)
  - Dimension – $[L^1]$
  - ❖ In case of a longitudinal wave wavelength is the distance between the centres of two successive compressions or rarefactions.
  - ❖ In case of a transverse wave wavelength is the distance between two successive crests or troughs.
- **FREQUENCY (f/n/η/ν) -:**
  - It is the number of complete vibrations performed by the particle in the medium in one second.
  - S.I. unit – hertz (Hz)
  - Dimension – $[T^{-1}]$
- **TIMEPERIOD (T) -:**
  - It is the time taken by the particle of the medium to complete one vibration.
  - S.I. unit – second (s)
  - Dimension – $[T^1]$
- **WAVE VELOCITY (v) -:**
  - It is the distance travelled by the wave form in one second.
  - S.I. unit – meter/second (m/s)
  - Dimension – $[L^1T^{-1}]$

RELATION BETWEEN TIME PERIOD AND FREQUENCY -:

- Let T be the time period and f be the frequency of the wave.
- 'f' number of complete vibrations are performed by the particle in one second

  So the time required to complete one vibration $= \frac{1}{f}$
- But this is called as the time period (T)
- So

$$\boldsymbol{time\ period\ (T) = \frac{1}{frequency(f)}}$$

- So time period and frequency are reciprocal of each other.

RELATION AMONG WAVE VELOCITY, FREQUENCY AND WAVELENGTH -:

- Let T be the timeperiod, λ be the the wavelength and f be the frequency of the wave.
- In a time 'T' (timeperiod), the wave travels a distance is equal to its wavelength (λ).

- So in one second the distance travelled by the wave $= \frac{wavelenght}{time\ period}$

$$= \frac{\lambda}{T}$$

But this called as wave velocity.

$$\text{So } wave\ velocity\ (v) = \frac{\lambda}{T}$$

$$v = \lambda f \qquad [f = \frac{1}{T}]$$

- So $$\boxed{\boldsymbol{wave\ velocity\ (v) = wavelength\ (\lambda) \times frequency\ (f)}}$$

ULTRASONICS -:

- Sound of frequency greater than the upper limit of audible range (20Hz – 20kHz) is termed as ultrasonic.
- It is also known as high frequency wave.

PROPERTIES -:

- It is longitudinal in nature.
- Propagation of ultrasonic wave through medium results in the formation of compression and rarefaction in the medium.
- It may have a frequency range of 2×10^4Hz to 2×10^9Hz.
- These are highly energetic waves.
- They travel with the speed of sound.

APPLICATIONS -:

- **Echo sounding –** Ultrasonic waves can be used to measure the depth of sea by echo sounding technic.
- **Flaw detection –** Ultrasonic waves are used to detect the presence of flaws or defects in the form of holes, porosity, cracks etc in the internal structure of the material.
- **Ultrasonic welding (cold welding) –** Two materials are welded together at room temperature by using ultrasonic waves.
- **Drilling holes –** Ultrasonic waves can be used for making holes in very hard and brittle materials like glass, diamond etc.
- **Medical use –** Ultrasonic waves are also used in medicals for diagnosis purpose (detection of any abnormal growth, tissue structure, tumors in the internal part of the body).